Classical Solutions of SU(3) Yang-Mills Theory and Heavy Quark Phenomenology

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Abstract. It is showed that potentials derived from classical solutions of the SU(3) Yang-Mills theory can provide confining potentials that reproduce the heavy quarkonium spectrum within the same level of precision as the Cornell potential.

In order to solve the classical Yang-Mills equations of motion, usually one writes an ansatz that simplifies the Euler-Lagrange equations and, hopefully, includes the relevant dynamical degrees of freedom. In [1] it was proposed a generalized Cho-Faddeev-Niemi-Shabanov ansatz for the gluon field, where the gluon is given in terms of two vector fields, $\hat{A}_\mu$ and Y^a_μ, and a covariant constant real scalar field n^a,

$$A^a_\mu \;=\; n^a \hat{A}_\mu \;+\; \frac{3}{2g} f_{abc} n^b \partial_\mu n^c \;+\; Y^a_\mu \tag{1}$$

with the constraints

$$D_\mu n^a \;=\; 0, \quad n^a Y^a_\mu \;=\; 0\,. \tag{2}$$

In [1] it was showed that the above decomposition of the gluon field is gauge invariant but not necessarily complete. In the weak coupling limit, $g \longrightarrow 0$, a finite gluon field requires either $n = 0$ or $\partial_\mu n = 0$. If $n = 0$, the gluon field is reduced to a vector field in the adjoint representation of SU(3) gauge group. For the other case, $\partial_\mu n = 0$, the gluon is writen in terms of the vector fields $\hat{A}_\mu$ and Y^a_μ and includes the previous solution as a particular case. Accordingly, a field such that $n \neq 0$ or $\partial_\mu n \neq 0$ does not produce a finite gluon field in the weak coupling limit and, in this sense, can be viewed as a nonperturbative field. Among this class of fields, the simplest parametrisation for the covariant scalar field[1] is $n^a = \delta^{a1}(-\sin\theta) + \delta^{a2}(\cos\theta)$. Then

$$A^a_\mu \;=\; n^a \hat{A}_\mu \;+\; \delta^{a3}\frac{1}{g}\partial_\mu \theta \;+\; \delta^{a8} C_\mu\,, \tag{3}$$

where $C_\mu = Y^8_\mu$. The classical Lagrangian and equations of motion are independent of θ and are abelian like in $\hat{A}_\mu$ and C_μ. Among the possible nonperturbative gluons given

[1] From the constraint equation $Dn = 0$ it follows that n^2 is constant. Our choice was $n^2 = 1$. A different value for the norm of n is equivalent to a rescaling of $\hat{A}$.

by (1), the simplest configuration has $\hat{A} = C = 0$. The coupling to the fermionic fields requires only the Gell-Mann matrix λ^3, decoupling the different colour components. This suggests, naively, that such a field is able to produce either confining, non-confining or free particle solutions for the quarks.

The classical equations of motion are independent of θ. However, a choice of a gauge condition, provides an equation for this field. For the Landau gauge, θ verifies a Klein-Gordon equation for a massless scalar field. Note that there is no boundary condition for θ, i.e. the usual free particle solutions of the Klein-Gordon equation are not the only possible ones. Indeed, writing $\theta(t,\vec{r}) = T(t)V(\vec{r})$, then

$$\frac{T''(t)}{T(t)} \;=\; \frac{\nabla^2 V(\vec{r})}{V(\vec{r})} \;=\; \Lambda^2 \;>\; 0\;, \tag{4}$$

$$T(t) \;=\; a e^{\Lambda t} \;+\; b e^{-\Lambda t}\;, \tag{5}$$

$$V(\vec{r}) \;=\; \sum_{l,m} V_l(r)\, Y_{lm}(\Omega)\;, \tag{6}$$

$$V_l(r) \;=\; \frac{\alpha_l}{\sqrt{z}} I_{l+1/2}(z) \;+\; \frac{\beta_l}{\sqrt{z}} K_{l+1/2}(z)\;, \tag{7}$$

where $z = \Lambda r$ and $I_{l+1/2}(z)$ and $K_{l+1/2}(z)$ are modified spherical Bessel functions of the $1^{\rm st}$ and $1^{\rm rd}$ kind[2]. The lowest multipole solution is

$$V_0(r) \;=\; A\,\frac{\sinh(\Lambda r)}{r} \;+\; B\,\frac{e^{-\Lambda r}}{r} \tag{8}$$

and the associated gluon field is given by

$$A^3_0 \;=\; \Lambda\left(e^{\Lambda t} - b e^{\Lambda t}\right)\, V_0(r)\,, \tag{9}$$

$$\vec{A}^3_0 \;=\; -\left(e^{\Lambda t} - b e^{\Lambda t}\right)\, \nabla V_0(r)\,. \tag{10}$$

From the lowest multipole solution one can derive a potential, which maybe suitable to describe heavy quarkonium. Indeed, assuming that quarks do not exchange energy, in the nonrelativistic approximation and leading order in $1/m$, the spatial function in A^3_0, $V_0(r)$, can be viewed as a nonrelativistic potential[3] and one can try to solve the associated Schrödinger equation. For the potential (8), the wave function goes to zero faster than an exponential for large quark distances,

$$\psi(\vec{r}) \;=\; \exp\left\{ \frac{-2}{\Lambda}\,\sqrt{\frac{2A}{m}}\;\exp\left(\frac{\Lambda r}{2}\right) \right\}. \tag{11}$$

As a first try to compute the heavy quarkonium spectra, we fixed A, B and Λ minimising the square of the difference between $V_0(r) + Constant$ and the Cornell potential [2] $V_{Cornell} = e/r + \sigma r$ ($e = -0.25$, $\sqrt{\sigma} = 427$ MeV) integrated between 0.2 fm and 1 fm.

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[2] Note that, by definition, the mass scale Λ is independent of a rescaling of the gluon field.

[3] The potential is $\sim 1/r$ for short distances and goes to infinity for large quark distances.

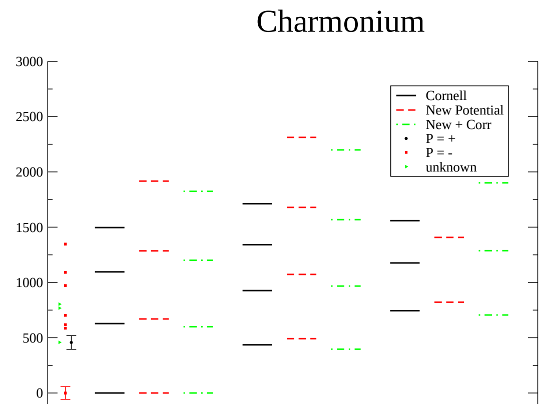

FIGURE 1. Heavy quarkonia spectra in MeV. The plots include the spin averaged experimental values.

This optimisation provides the following parameters $A = 5.4$, $B = -1.0$, $\Lambda = 281$ MeV, $Constant = -1190$ MeV; for these values $-24MeV \leq V_{Cornell} - (V_0 + Constant) \leq 64$ MeV in the integration range considered. Then, we can compare the Schrödinger equation spectrum for the charmonium ($m_c = 1.25$ GeV) and for the bottomonium ($m_b = 4.25$ GeV) for the two potentials. The spectrum for the new potential shows an equal level spacing for both the charmonium and bottomonium spectra. If the V_0 charmonium spectrum is quite close to the Cornell spectrum, the botomonium shows clear deviations; see figure 1. The differences are the result of overestimating the strengh of $V_0(r)$ for smaller distances. Indeed, one can improve our potential linearising the full QCD equations around the above configuration. To lowest order, this is equivalent to add a term like k/r to V_0. Computing k perturbatively[4] adjusting the $M[(1P)] - M[(1S)]$ bottomonium mass difference, gives $k = 0.2448251$. The heavy quarkonia spectra, including this correction, is given in figure 1.

In conclusion, classical configurations seem to be able to produce a spectra close to the Cornell potential. Hopefully, this is an indication that these configurations can be of help to understand strong interaction physics. Of course, there are a number of issues that need to be further investigated (definition of the potential parameters, inclusion of time dependence, decay rates). We are currently working on these topics and will provide a report soon.

REFERENCES

1. O.Oliveira, R. A. Coimbra, hep-ph/0305305
2. See G. S. Bali, Phys. Rep.343(2001)1-136 [hep-ph/0001312] and references therein.

[4] For each stationary, the shift in energy due to this term is compatible with a perturbative treatment. Corrections are clearly below 10-20%.